Measures of prestige

Prestige measures are usually computed for directed networks only, since for this measures the direction is important property of the relation.

Example: Persons, who are chosen as friends by many others have a special position – prestige in the group.

Prestige becomes salient especially if positive choices are not reciprocated, for instance, if everybody likes to play with the most popular girl or boy in a group, but s/he does not play with all of them. Or, in the case of sentiments, if people tend to express positive sentiments towards prestigious persons but receive negative sentiments in return. In these cases, social prestige is connected to social power and the privilege not to reciprocate choices.

Input degree

As already mentioned, input (output) degree is one of the measures of prestige. According to meaning of relation they represent support or influence. In the following we will suppose that we analyze the relation of positive selection (e.g., *likes*, *votes for*,. . .).

Degree is a very restricted measure of prestige because it takes into account direct choices only. With popularity it does not matter whether choices are received from people who are not chosen often themselves, or from people who are popular themselves. The overall structure of the network is disregarded.

Influence domain

Several efforts have been made to extend prestige to indirect choices. The first idea which comes to mind is to count all people by whom someone is nominated directly or indirectly, that is, with or without go-betweens. The index has been called the *influence domain* of a vertex because structural prestigious people are thought to influence the people who regard them as their leaders.

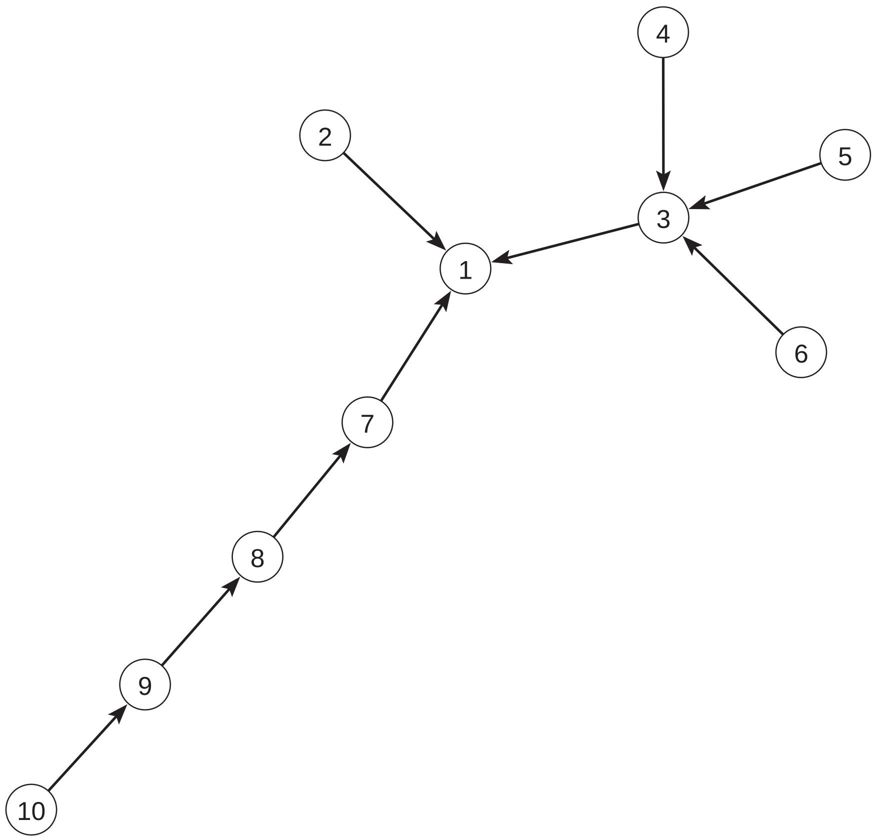
The influence domain of a vertex in a directed network is *the number or proportion of all other vertices which are connected by a path to this vertex*.

Proportion of vertices in influence domain is computed by divided number of vertices in influence domain by $n - 1$ (number of all other vertices).

It is easy to see that this measure is useful only if the network is not strongly connected. (otherwise there will be all other vertices in influence domain of every vertex). Instead of influence domain, for strongly connected networks (input) closeness centrality is used.

Drawback of the measure

As a measure of prestige, the influence domain of a vertex does not distinguish between direct and indirect choices, which is not completely desirable. Usually, we consider direct choices more prestigious than indirect choices. A choice contributes less to prestige if it is mediated by a longer chain of intermediaries.



All other vertices are in influence domain of person 1. But choices by actors 2, 3, and 7 are more important to person 1 than indirect choices by 4, 5, 6, and 8. Individuals 9 and 10 contribute even less to the prestige of 1.

In order to let direct choices contribute more to prestige than indirect choices, we can weight each choice by its path distance to the selected vertex. A higher distance yields a lower contribution to the prestige of another vertex. We can simply compute mean distance of a vertex from vertices in its influence domain, e.g., mean distance to vertex 1 is 2.0 (18/9).

However, this index has an important limitation. In figure, mean distance from vertices in the influence domain is minimal for vertices 3 and 9, because they are directly chosen by all vertices in their influence domains: vertices 4, 5, 6, and 10 respectively.

In the entire network, however, these vertices are clearly not more prestigious than vertex 1. Their small influence domains are responsible for low average distance.

Therefore, average distances must be combined with the size of influence domains in order to make sense.

Proximity prestige

We compute ***proximity prestige (PP)*** of selected vertex by dividing the influence domain of a vertex (expressed as a proportion) by the average distance from all vertices in the influence domain.

A larger influence domain and a smaller distance yield a higher proximity prestige score. Maximum proximity prestige is achieved if a vertex is directly chosen by all other vertices. Then, the proportion of vertices in the influence domain is 1 and mean distance from these vertices is 1, so proximity prestige is 1 (1 divided by 1). Vertices without influence domain get minimum proximity prestige by definition, which is 0.

In figure, all vertices at the extremes of the network (2, 4, 5, 6, and 10) have empty influence domains, hence they have proximity score 0.

The influence domain of vertex 9 contains vertex 10 only, so its size is 1 out of 9 (0.11). Average distance within the influence domain of vertex 9 is 1, so the proximity prestige of vertex 9 is 0.11 divided by 1.

Vertex 1 has maximal influence domain, since it can be reached by all 9 vertices (a proportion of 1.00). Average distance is 2.0, so proximity prestige amounts to 1.00 divided by 2.0, which is 0.5. All remaining vertices have proximity scores lower than vertex 1, but higher than vertex 9.

In the table influence domain, its proportion, average distance and proximity prestige for all vertices are given.

vertex	ID	$ID_r = \frac{ID}{9}$	$\bar{d}$	$PP = \frac{ID_r}{\bar{d}}$
1.	9	1.00	2.00	0.500
2.	0	0.00	?	0.000
3.	3	0.33	1.00	0.333
4.	0	0.00	?	0.000
5.	0	0.00	?	0.000
6.	0	0.00	?	0.000
7.	3	0.33	2.00	0.167
8.	2	0.22	1.50	0.148
9.	1	0.11	1.00	0.111
10.	0	0.00	?	0.000

See network infl.net.

Special example: If network is strongly connected, proximity prestige is equal to input closeness centrality.

Another example

Data about import and export among 22 countries are available.

Network is available as wass749a.net – Trade of basic manufactured goods between countries, Wasserman & Faust, 749.

An arc from country x to country y means, that country x exports to country y. In this case centrality measures tell us which countries are important exporters and which are important importers.

In this case the most suitable measure of prestige is degree (input or output).

The network is strongly connected, therefore proximity prestige is equal to input closeness centrality (C_{Cin}).

No.	Country	C_{Dout}	C_{Cout}	C_{Din}
1.	Alg	0.190	0.553	0.619
2.	Arg	0.619	0.724	0.476
3.	Bra	0.905	0.913	0.524
4.	Chi	0.905	0.913	0.714
5.	Cze	0.905	0.913	0.619
6.	Ecu	0.095	0.525	0.429
7.	Egy	0.429	0.636	0.571
8.	Eth	0.095	0.525	0.476
9.	Fin	0.953	0.955	0.714
10.	Hon	0.048	0.512	0.429
11.	Ind	0.667	0.750	0.667
12.	Isr	0.524	0.677	0.476
13.	Jap	*1.000	*1.000	*0.810
14.	Mad	0.048	0.512	0.286
15.	NZ	0.524	0.677	0.667
16.	Pak	0.524	0.677	0.667
17.	Spa	0.952	0.955	*0.810
18.	Swi	*1.000	*1.000	0.714
19.	Tai	0.619	0.724	0.714
20.	Uk	0.952	0.955	0.762
21.	US	*1.000	*1.000	*0.905
22.	Yug	0.810	0.840	0.714
	Central	0.412	0.495	0.307

Hubs and authorities

Look at another two measures of prestige, which are especially useful in the case of WWW (directed network of home pages).

In directed networks we can usually identify two types of important vertices: hubs and authorities. Each home page describes something (is an *authority*) and because of that other pages point to it. But on the other hand each page points to some other pages (is a *hub*).

Vertex is a *good hub*, if it points to many *good authorities*, and is a *good authority*, if it is pointed to by many *good hubs*.

See: Kleinberg, Jon M.: Authoritative Sources in a Hyperlinked Environment. *Proceedings of the 9th ACM-SIAM Symposium on Discrete Algorithms.* Edited by Howard Karloff (SIAM/ACM-SIGACT, 1998).

For each vertex v we computed two weights $x_v, y_v \in [0, 1]$, which tell how good authority and hub us a given vertex. We say that vertex v is better authority than vertex u, if $x_v > x_u$.

Weights are computed according to network by solving the eigenvector problem of matrices AA^T (hubs) and A^TA (authorities).

Example

Our network example describes the 32 soccer teams which participated in the World Championship in Paris, 1998. Players of the national team often have contracts in other countries. This constitutes a players market where national teams export players to other countries. Members of the 32 teams had contracts in altogether 35 countries. Counting which team exports how many players to which country can be described with a valued, asymmetric graph. The graph is highly asymmetric: some countries only export players, some countries are only importers (data collected by Lothar Krempel).

The relation *player from country x plays in country y* can be represented by a directed network, where value a on line between x and y means, that there play a players from country x in country y.

Network analysis

- *Analysis of degrees*
  - There exist countries that are only exporters: Yugoslavia, Romania, Norway, Nigeria...
  - There exist countries that are only importers: Spain, France, Turkey, Greece, GBR...
  - Spain imports players from the highest number of different countries (18); following by Italy and Germany importing from 16 countries.
  - Nigeria exports players into the highest number of different countries (10).
- *Hubs and Authorities*

  Authorities are countries having good leagues, hubs are countries having good players.
  - *Hubs:* Yugoslavia, Norway, Argentina, Romania, Croatia, Denmark... Yugoslavia (0.41) and Norway (0.38) are much more important hubs than the others (Argentina - 0.28).
  - *Hubs and authorities:* The Netherlands, Scotland, Brazil, Austria...
  - *Authorities:* Spain (0.58), GBR (0.54), Italy (0.45),

Germany (0.31), France (0.16), Turkey (0.16)...

Countries are written in descending order according to hubs/authorities weights.

Comparing degree centrality to hubs and authorities two questions arise:

1. *Why is Nigeria not an important hub, although it exports to the highest number of different countries?*

   There are two reasons:

   - Degree centrality used counts only number of different countries where players are exported and not how many players, while this information is used when computing hubs and authorities. If we count how many players are imported (*valued input degree*) we get: Spain (54), GBR (50), Italy (42), Germany (39),... Ranking according to the number of exported players (*valued output degree*): Norway (22), Yugoslavia (21), Nigeria (21), Cameroon (20),... We see that Nigeria is not in the first place any more.

     Valued degree is computed by multiplying network with vector containing only 1's.
   - Even more important reason is that Nigeria exports much smaller proportion of its players to the best

leagues (authorities) than Yugoslavia and Norway, for example: Into the best leagues (Spain, Italy, Germany, France and GBR)

- Nigeria exports only 10 of its 21 exported players (48%);
- Yugoslavia exports 19 of its 20 exported players (90%);
- Norway exports 17 of its 22 exported players (77%).

2. *Why is Argentina among hubs although it does not export many players (only 13)?*

   Argentina exports this 13 players only to two countries, but these two countries have the best leagues (are authorities): it exports 4 players to Spain and 9 players to Italy.

Measures of prestige in Pajek

We already know how to compute ordinary input or output degrees.

Computing valued degrees is done by:
Net/Vector/Summing up Values of Lines/Input for weighted input degree, and
Net/Vector/Summing up Values of Lines/Output for weighted output degree. Result is a new vector.

For computing influence domain and proximity prestige we call: Net/Partitions/Domain/Input. The results are:

- partition containing number of vertices in influence domain
- vector containing proportion of vertices in influence domain
- vector containing average distances from vertices in influence domain

We compute proximity prestige by dividing the first obtained vector by the second (Vectors/Divide First by Second).

Hubs and authorities are computed in
Net/Vector/Important Vertices/1-Mode: Hubs/Authorities
We get additional question: How many hubs and how many authorities are marked in the partition. Results are:

- Partition, where value 1 (yellow) means, that the vertex is a good authority, value 2 (green) means, that the vertex is a good authority and a good hub, and value 3 (red) means, that the vertex is a good hub.
- Vector with *Hub Weights*, larger value means better hub.
- Vector with *Authority Weights*, larger value means better authority.

Be careful: Algorithm supposes that values on lines represent similarities (larger value means more important choice). In the case that it is the opposite, we must set all values to 1, otherwise results will be wrong. This can be done by saving the network in the form of binary matrix (Pajek Binary Matrix), where all nonzero values are changed to 1. Then we read the new file again.

Comparison of prestige measures

Four prestige measures were mentioned:

- input degree;
- input closeness, or proximity prestige, if network is not strongly connected;
- betweenness;
- authorities.

Some of these measures consider prestige in a similar way, some are quite different. For comparing the results of different measures we can use *Pearson correlation coefficient*, which is available in Vectors/Info, but we must first select both vectors.

In general we find out that most measures are very similar except betweenness, which measures prestige in a quite different way.

Examples

1. In network exclique.net (whom will you ask for help) compute proximity prestige and hubs and authorities.

2. Compute all measures of prestige, which are possible to compute in directed network sampson.net (liking among the monks). Draw the picture with sizes of vertices proportional to the computed prestige measure.

3. Compute all measures of centrality and prestige, which are possible to compute in directed network wass749a.net (import/export among 22 countries).

4. Compute input and output degrees, valued input and output degrees and hubs and authorities in network football.net (export of soccer players among 35 countries).